Universality of the contact process with random dilution

Marcelo M. de Oliveira

E-mail: mancebo@fisica.ufmg.br

Silvio C. Ferreira

E-mail: silviojr@ufv.br
Departamento de Física, Universidade Federal Viçosa, 36571-000, Viçosa, MG, Brazil

Abstract. We present quasi-stationary simulations of the two-dimensional contact process with quenched disorder included through the random dilution of a fraction of the lattice sites (these sites are not susceptible to infection). Our results strongly indicate that the static exponents are independent of the immunization fraction. In addition, the critical moment ratios $m = \langle\rho^2\rangle/\langle\rho\rangle^2$ deviate from the universal ratio $m = 1.328$, observed for the non-dilluted system, to smaller values due to rare favorable regions which dominate the statistics.

PACS numbers: 05.70.Ln, 89.75.Da, 89.75.Hc, 05.70.Jk

Submitted to: *J. Stat. Mech.*

1. Introduction

Absorbing-state phase transitions, i.e, transitions from a fluctuating phase to an absorbing (trapped) state, are related to several nonequilibrium critical phenomena [1, 2, 3, 4] such as chemical catalysis [5], interface growth [6], epidemic spreading [7], etc. The study of such transitions in spatially extended systems has been experimenting an ongoing interest, strengthened by recent experimental confirmations of absorbing-state phase transitions in a liquid crystal system [8], and in a sheared colloidal suspension [9]. Notwithstanding a complete classification of their critical behavior is still missing, it has been conjectured [10, 11] that models with a positive one-component order parameter, short-range interactions and deprived of additional symmetries or quenched disorder belong generally to the universality class of directed percolation (DP), which is considered the most robust universality class of the absorbing-state phase transitions.

The contact process (CP) [12] is one of the simplest and most studied models of the DP universality class. Of particular interest is how spatially quenched disorder affects its critical behavior [13, 14]. Quenched disorder, in the form of impurities and

defects, plays an important role in real systems, and may be responsible for the rarity of experimental realizations, in spite of the ubiquity of the DP class [15].

The so-called Harris' criterion [17] states that quenched disorder is a relevant perturbation, from the field-theoretical point of view, if

$$d\nu_\perp < 2, \tag{1}$$

where d is the dimensionality and $\nu_\perp$ is the correlation length exponent of the pure model (In DP this inequality is satisfied in all dimensions $d < 4$, since $\nu_\perp = 1.096854(4)$, $0.734(4)$ and $0.581(5)$, for $d = 1$, 2 and 3, respectively [18, 19, 20]). The first numerical studies of the CP with quenched disorder, introduced by the means of a random deletion of sites [21, 22] or bonds [23] (dilution) or by random random spatial variation of the control parameter [24], confirmed that the disordered system does not belong to the DP class [21], and also revealed that the critical spreading is logarithmic, not a power law [22]. In the subcritical regime, a Griffits-phase, with critical dynamics dominated by nonuniversal power laws was also reported [22, 13]. However, the contact process in a Voronoi-Delaunay lattice, which has an intrinsic quenched disorder in the distribution connectivity, belongs to the DP class [25].

Hooyberghs et al. [26, 27], employed a strong-disorder renormalization group approach to conclude that the unusual critical behavior of the disordered system can be related to the random transverse-field Ising model, for sufficiently strong disorder. At such infinite-randomness fixed point, the scaling is activated, i.e, the temporal and spatial correlation lengths ($\xi_\parallel$ and $\xi_\perp$, respectively) are related by

$$\ln \xi_\parallel \sim \xi_\perp^\psi \tag{2}$$

where ψ is a universal exponent. For weak disorder they found nonuniversal critical exponents depending on the disorder strenght.

More recently, Votja and Lee [28] used this activated dynamic scaling to show that the interplay between geometric criticality and dynamic fluctuations leads to a novel universality class, with the exponent ψ equals to the fractal dimension of the critical percolation cluster of the diluted lattice. Previous numerical studies of the one-dimensional contact process with quenched spatial disorder, performed by Votja and Dickinson [29] also supported the activated exponential dynamical scaling at the critical point. Moreover, they found evidences that this critical behavior turns out to be universal, even for weak disorder. Novel strong disorder renormalization group calculations in a very recent paper by Hoyos [30] predict that the system is driven to the infinite-randomness fixed point, independently of the disorder strength.

Thus, despite of a deeper understanding of the effects of quenched disorder in the critical contact process achieved in the last decade, a certain controversy remains: Do the static critical exponents change continuously with the degree of disorder [22, 13, 26], or do they change abruptly to the values in the strong disorder limit corresponding to the universality class of the random transverse Ising model [29, 28]? Obtaining the static exponents is a hard numerical task because at criticality the infinite disorder fixed point is characterized by a ultra-slow dynamics, $\rho(t) \sim \ln(t)^{-\beta/(\nu_\perp\psi)}$, leading to a unusual long relaxation towards the quasi-stationary (QS) values. Thus, in a tentative to shed some light into this issue, we present results of extensive large-scale QS simulations [31] of the two-dimensional diluted contact process.

The balance of this paper is organized as follows: In the next section we review the definition of the contact process and describe the simulation method. In Sec. III we present our results and discussion, and Sec. IV is devoted to draw some conclusions.

2. Model and methods

The contact process is a stochastic interacting particle process in which the particles lie on the sites of a d-dimensional hypercubic lattice. Each site can be vacant or occupied. An empty site becomes occupied at a rate $\lambda n/d$, where n is the number of its occupied nearest-neighbors, while occupied sites become vacant at unitary rate [12, 1]. For a certain critical value $\lambda = \lambda_c(d)$, the model exhibits a continuous phase transition from an active state (with a positive density of sites occupied) to an absorbing configuration with all sites vacant, since none particle can be created from the vacuum.

In the simulation we employ the usual scheme [1]: with probability $p = \lambda/(1+\lambda)$, one nearest neighbor j of the selected site i is chosen at random and occupied if the site j is vacant. With complementary probability $q = 1/(1+\lambda)$ the particle at site i is annihilated. At each step, the time is increased by $\Delta t = 1/N_{occ}$, where N_{occ} is the total number of occupied sites. Moreover, occupied sites are sequentially selected at random from a list constantly updated, in order to improve efficiency.

In the original diluted contact process (DCP) [21, 22], a quenched disorder is introduced by labeling each site as diluted or nondiluted with probabilities x and $1-x$, respectively. In the present work, we mark as diluted exactly a fraction Γ of the sites, in order to avoid undesirable extra fluctuations. Thus, DCP is the CP model restricted to the nondiluted sites since those diluted are never occupied. Notice that, for large systems, the dilution process used in this work is equivalent to those used in reference [22].

Stationary analysis nearby the critical point of systems with transitions to absorbing configurations are hard to be done due to very strong finite size effects. Indeed, the unique real stationary state of finite systems is the absorbing configuration. A common alternative to avoid this difficulty is to restrict the averages to the surviving samples and proceed a finite size analysis. This procedure [1], involves careful scrutiny in the data analysis which is not always free of ambiguities or misinterpretations [4]. In order to circumvent such difficulties, we employ a simulation method that yields quasistationary (QS) properties directly, the QS simulation method [31]. The method is based in maintaining, and gradually updating, a list of M configurations visited during the evolution; when a transition to the absorbing state is imminent the system is instead placed in one of the saved configurations. Otherwise the evolution is exactly that of a conventional simulation. By this procedure one obtains an unbiased sampling of the quasistationary distribution of the process.

The simulations were performed as follows. Firstly, the list of configurations is incremented whenever the time increases by a unity up to a list with $M = 1000$ configurations is achieved. Secondly, a configuration of the list is randomly chosen and replaced by the current one with a given probability p_{rep}. We used a large value of $p_{rep} = 0.5$ for a initial relaxation period, more precisely for $t < L^{1.5}$ where L is the linear system size, in order to speed up the erasing of the memory of the initial condition. Also, $p_{rep} = 0.005$ was adopted for the remaining of the simulation.

3. Results and Discussion

We studied lattice sizes varying from $L = 20$ to $L = 640$, averaging over 200 to 300 independent realizations of disorder, of duration up to $t = 2 \times 10^8$. Averages were taken after a relaxation time $t_r = 10^8$. The larger sample sizes and longer run times apply to the larger L values.

The first step in analyzing our results is to determine, for each dilution value studied, the critical creation rate $\lambda(\Gamma)$. For this purpose we study the number of active particles, $n(t)$ via spreading analysis. The critical value λ_c is then defined as the smallest λ supporting asymptotic growth (vide Fig.1). This criterion avoids misinterpretations associated to the effects due to the Griffiths phase, in which power laws in $n(t)$ are observed [22].

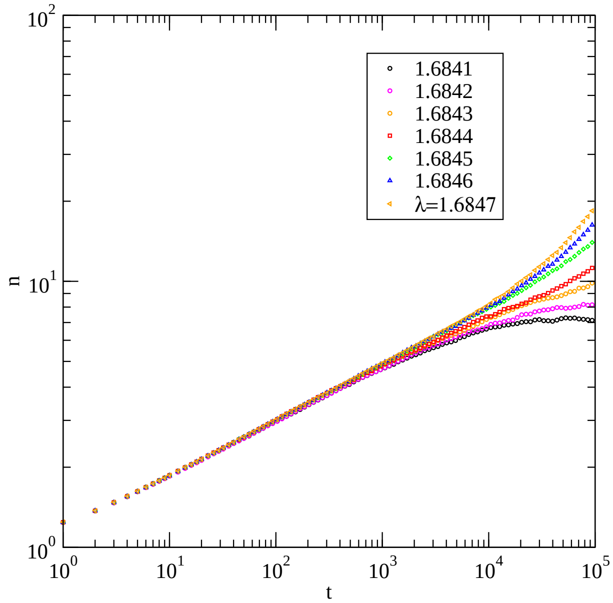

Figure 1. Spreading of activity from a single seed. Dilution rate: $\Gamma = 0.02$.

In Fig. 2 we show a log-log plot of the critical quasistationary density of active sites ρ, as function of L for dilutions ranging from 0.02 to 0.30. At criticality, such quantity decays as a power law, $\rho \sim L^{-\beta/\nu_\perp}$. (This permits us to check the critical values λ_c obtained from the spreading analysis). The values for the critical exponent $\beta/\nu_\perp$, obtained from linear least-squares fits to the last four points of the data are shown in Table I. Our results suggest that the critical exponent ratio $\beta/\nu_\perp$ is *independent* of the amount of dilution, with $\beta/\nu = 0.95(2)$, at least for dilutions $\Gamma \geq 0.05$.

Table 1. Exponent ratio $\beta/\nu_\perp$ for several dilution values. $^{a)}$Present work using $L \leq 640$. $^{b)}$Taken from [22] where $L \in [8, 128]$ was used.

Γ	λ_c	$^{a)}\beta/\nu_\perp$	$^{b)}\beta/\nu_\perp$
0	1.64874(1)	0.797(2)	0.80
0.02	1.6844(1)	0.87(1)	0.83
0.05	1.7410(1)	0.94(2)	0.82
0.10	1.84640(5)	0.96(2)	0.85
0.20	2.1075(2)	0.95(2)	0.86
0.30	2.473(1)	0.95(1)	0.92

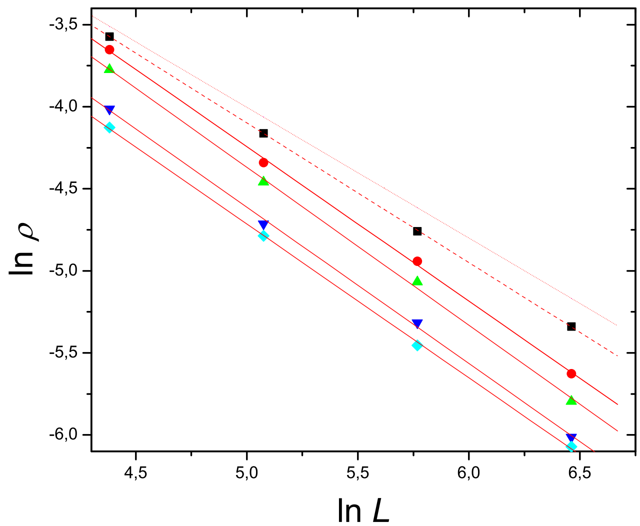

Figure 2. Quasistationary densities of active sites versus system size L. From top to bottom: nondiluted CP (dotted line), $\Gamma = 0.02$ (dashed), and $\Gamma = 0.05, 0.10, 0.20$ and 0.30.

Now we turn to the dynamical exponent $z = \nu_{\|}/\nu_{\perp}$. In the nondiluted CP, the lifetime of the QS state (which we take as the mean time between two attempts to absorbing in the QS simulation), follows $\tau \sim L^z$. On the other hand, in the activated dynamical scenario, such power-law scaling is replaced by $\ln \tau \sim L^{\psi}$, with ψ being an universal exponent. In other words, the critical exponent z is formally infinity in this scenario. Early works [22, 26] found a nonuniversal power-law behavior, with an exponent z varying continuously in direction to the strong disorder values. Otherwise, our results, shown in Fig.3, reveal that at criticality the lifetime behavior is not a power law, clearly diverging with an increasing slope for all $\Gamma \geq 0.05$. This suggest that the activated scaling emerge even for weak disorder, enforcing the universality hypothesis. Furthermore, applying the activated scaling to the the data for the last four points for highest dilution, $\Gamma = 0.30$ furnishes the value of $\psi = 0.48(7)$, consistent with the values of the exponent ψ in the range $0.42 < \psi < 0.50$ found in the literature for the random transverse Ising model [32, 33]. For weaker disorder, our results do not permit to distinguish between a scenario with continuously varying ψ to a possible crossover to a universal value at larger system sizes, as predicted by the activated dynamics.

Another consequence of the activated dynamics scenario is that the distribution of the observables become broader, implying that the averages are dominated by the rare events in which the process is locally supercritical. Thus, some quantities such as moment ratios of the order parameter (which converges to universal values in the nondiluted CP [34]) exhibit non-self-averaging properties, in the sense that they do not converge to limiting values even when $L \to \infty$ [22, 35, 36]. This is exemplified in Fig.

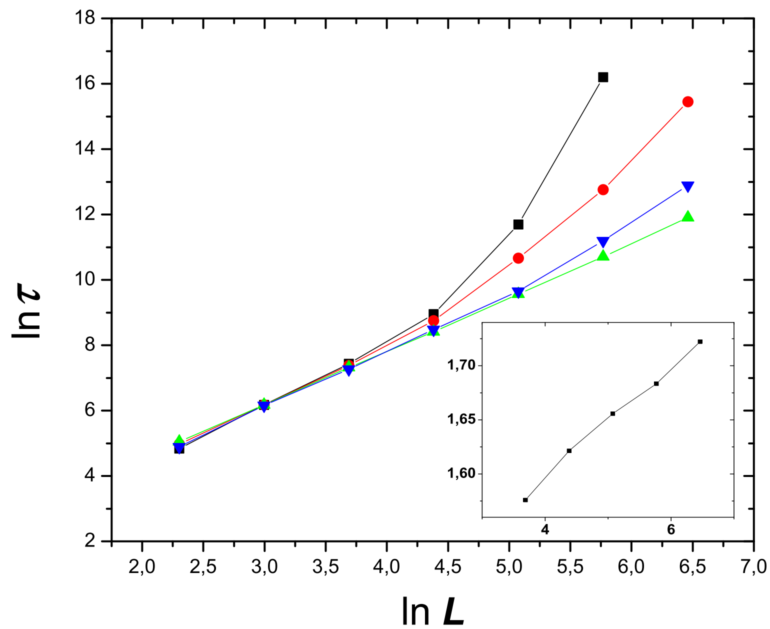

Figure 3. Quasistationary lifetime τ versus system size L. Dilution rates: $\Gamma = 0.05, 0.10, 0.20, 0.30$, from bottom to top. Inset: $d\Gamma/dL$ for $\Gamma = 0.05$.

4 where the moment ratio $m = \langle\rho^2\rangle/\langle\rho\rangle^2$ is plotted as a function of the system size L. The effects of the rare 'favorable' regions dominate the statistics, and the moment ratio is drifted from the DP value of $m \sim 1.328$ to smaller values (in the favorable regions the process is locally supercritical, and $m \to 1$ in the limit $\lambda \to \infty$). We observe that for high dilutions the effects of the rare regions become observable even for modest system sizes, while for low dilution the effect only appear at considerable larger system sizes. Notice that for the dilution $\Gamma = 0.02$ these effects are not evident for the system sizes we used, what may justify the difference in the exponent ratio $\beta/\nu_\perp$ for the smallest value of Γ.

4. Conclusions

We performed extensive large-scale simulations of the two-dimensional contact process with dilution. The dilution is known to change the critical behavior of the contact process. Our results indicate that the novel static exponents do not depend on the amount of dilution, and we present numerical evidences that the apparent nonuniversality observed in early works was due to finite-size effects. Our findings are in agreement with recent simulational results for the one-dimensional contact process with quenched disorder [28], and with strong disorder renormalization group results [30]. On the other hand, our results cannot exclude a nonuniversal variation

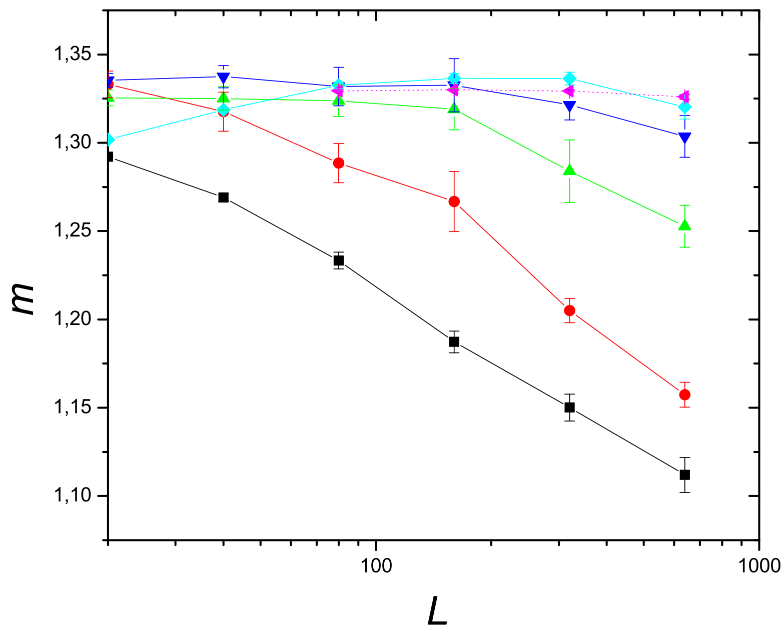

Figure 4. Moment ratio m for dilutions $\Gamma = 0.30, 0.20, 0.10, 0.05$ and 0.02 from bottom to top. The dashed line represents the nondiluted $\Gamma = 0.00$ case.

of the exponent ψ with the disorder strength. Finally, the critical moment ratios $m = \langle\rho^2\rangle/\langle\rho\rangle^2$ deviate from the the universal ratio $m = 1.328$ of the non-diluted system to smaller values, due to rare favorable regions which dominate the statistics.

Acknowledgments

We thank Ronald Dickman for helpful discussions and for pointing us reference [30]. This work was partially supported by CNPq and FAPEMIG Brazilian agencies.

[1] Marro J and Dickman R, 1999 *Nonequilibrium Phase Transitions in Lattice Models* (Cambridge University Press: Cambridge).
[2] Hinrichsen H, 2000 Adv. Phys. **49** 815.
[3] Ódor G, 2004 Rev. Mod. Phys **76** 663.
[4] Lubeck S, 2004 J. Mod. Phys. B **18** 3977.
[5] Ziff R M , Gulari E, and Barshad Y, 1986 Phys. Rev. Lett. **56** 2553.
[6] Tang L H and Leschhorn H, 1992 Phys. Rev. A **45** R8309.
[7] Bartlett M S, 1960 *Stochastic Population Models in Ecology and Epidemiology* (Methuen: London).
[8] Takeuchi K A, Kuroda M, Chaté H, and Sano M, 2007 Phys. Rev. Lett. **99** 234503.
[9] Corté L, Chaikin P M, Gollub J P, and Pine D J, 2008 Nature Physics **4** 420.
[10] Janssen H K, 1981 Z. Phys. B **42** 151.
[11] Grassberger P, 1982 Z. Phys. B **47** 365.
[12] Harris T E, 1974 Ann. Probab. **2** 969.
[13] Cafiero R, Gabrielli A, and Munoz M A, 1998 Phys. Rev. E **57** 5060.
[14] Webman I, ben-Avraham D, Cohen A, and Havlin S, 1998 Phil. Mag. B 77, 1401.

[15] Hinrichsen H, 2000 Braz. J. Phys. **30** 69.
[16] Langhorne J, Ndungu FM, Sponaas A, and Marsh K, 2007 Nature Immun. **9** 725.
[17] Harris A B, 1974 J. Phys. C **7**, 1671.
[18] Jensen I, 1999 J. Phys A **32** 5233.
[19] Voigt C A and Ziff R M, 1997 Phys. Rev. E **56** R6241.
[20] Jensen I, 1992 Phys. Rev. A **45** R563.
[21] Noest A J, 1986 Phys. Rev. Lett. **57** 90;
Noest A J, 1988 Phys. Rev. B **38** 2715.
[22] Moreira A G and Dickman R, 1996 Phys. Rev. E **54** R3090;
Dickman R and Moreira A G, 1998 Phys. Rev. E **57** 1263.
[23] Dahmen S R, Sittler L, and Hinrichsen H, 2007 J. Stat. Mech P01011.
[24] Bramson M, Durrett R, and Schonmann R, 1991 Ann. Prob. **19**, 960.
[25] de Oliveira M M, Alves S G, Ferreira S C, Dickman R, 2008 Phys. Rev. E **78**, 031133.
[26] Hooyberghs J, Iglói F, and Vanderzande C, 2003 Phys. Rev. Lett. **90** 100601.
[27] Hooyberghs J, Iglói F, and Vanderzande C, 2004 Phys. Rev. E **69** 66140.
[28] Vojta T and Lee M Y, 2006 Phys. Rev. Lett. **96**, 035701.
[29] Vojta T and Dickison M, 2005 Phys. Rev. E **72**, 036126.
[30] Hoyos J A, 2008 Phys. Rev. E **78**, 032101.
[31] de Oliveira M M and Dickman R 2005, Phys. Rev. E **71** 016129;
Dickman R and de Oliveira M M, 2005 Physica A **357** 134.
[32] Motrunich O et al., 2000 Phys. Rev. B **61** 1160.
[33] Karevski D et al., 2001 Eur. J. Biochem. **20** 267.
[34] Dickman R and Kamphorst Leal da Silva J, 1998 Phys. Rev. E **58** 4266.
[35] Wiseman S and Domany E, 1995 Phys. Rev. E **52** 3469.
[36] Aharoni A and Harris A B, 1996 Phys. Rev. Lett. **77** 3700 .